

## Al-Khwarizmi's contribution to making computational-algorithmic tradition in the era of european renaissance



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**Abstract:** The article tells about the scientific work of the great Uzbek scientist Muhammad al-Khwarizmi and his historical merits in the world science. Algorithms are one of the hundred names of al-Khwarizmi. The article details the evolution of this term. The article also shows the role of algorithmic ideas and the al-Khwarizmi scientific school in becoming of well-known European mathematicians such as Leonardo Pisansky, Paccioli and many other Renaissance scientists.

**Key words:** Al-Khwarizmi, algorithm, Central Asia, mathematics, arithmetic treatise, world civilization.

### Introduction

As is known Uzbekistan is one of the cradles of the world science in the history of the development of world civilization. It was Uzbek land which had given the world such prominent thinkers of the Oriental and Western sciences as Beruni, Farabi, Al-Khwarizmi, Ulugbek and many others.

The achievements of mathematics and computational - algorithmic sciences of Central Asia during IX-XV cc. have left the lasting trace in the development of the world scientific thought. In the works of Central Asian mathematicians of paramount importance was the development of computational techniques. The zeal towards the perfection of these techniques was revealed in the works of Uzbek classicists of science which served as the basis for European mathematics and affected greatly the development of the world computational-algorithmic trend.

The works of prominent mathematicians of Central Asia and Uzbekistan enjoyed an invariable authority in the Orient in XI-XV cc. and later in Europe.

### Literature review

Al-Khwarizmi's contribution had been studied and commented by the scientists of subsequent generations being the guidelines for research in various fields of mathematics.

This issues in the Republic of Uzbekistan are considered in the publications of scientists of Tashkent State University of Economics, M. Ulugbek National University of Uzbekistan (Uzbekistan), Plekhanov G.V. Russian University of Economics (Russian Federation), Coordination and Methodological Center of the Uzbekistan's contemporary history under the Academy of Sciences of Uzbekistan (Uzbekistan).

Of special interest were scientific works of the outstanding Uzbek scientist Muhammad ibn Musa al-Khwarizmi who was at the source of contemporary mathematics and computer science, astronomy, geography; - he made a weighty contribution to the

treasure-house of the world science: no wonder a noted American scholar of science George Sarton has named the whole first part of IX c. the epoch of Al-Khwarizmi as “the most famous mathematician of his time and taking into account all the circumstances – one of the greatest of all times”[4,p.8].

German scientist K.Vogel named Khwarizmi “the teacher of the West not only in algebra but also in his introduction of new Indian figures and methods of calculations”[16, p.89]. K.Vogel continues: ... arithmetic’s beginning with words “Dixit Algorizmi” lay down the foundation of the rapid growth of mathematical knowledge which had occurred in Western Europe during XVI-XVII cc.[9].

Well-known American scholar of Central Asia Frederick Starr also argues that, “In mathematics, Khwarazmi was the first to elaborate a theory of equations solvable through radicals, which can be applied to the solution of a variety of arithmetical and geometrical problems. The result was a book, Algebra, that gave its name to the field; the term algorithm is a corrupted form of his own name. Khwarazmi advanced the field of spherical astronomy and did more than anyone else to popularize the decimal system that had been invented in India”.

The publications of the following authors are devoted to theoretical understanding of the issues of Khwarazmi contribution to making computational-algorithmic tradition in the era of european renaissance: G.P.Matvievskaia [14], A.P.Yuschkevich [18], P.G.Bulgacov [6].

The issues of the need to develop research and educational work, to promote international cooperation, to improve training in mathematics are considered in the publications of the Academician of the Academy of Sciences of Uzbekistan K.Kh. Abdurakhmanov, N.K. Zokirova, R.K. Kabulov, A.F. Fayzullaev, Sh.Nasirov [12].

### **Research methodology**

In fact, the arithmetic treatise of Khwarizmi “Kitab hisab al adad al hindi” (“the book on counting by Indian figures”) has played a leading role in history not only in arithmetics, mathematics and its computational – algorithmic trend, but also the world culture at large as the main source of propagation of positional number notation. It replaced throughout considerably less elaborated alphabetic system of calculation used by Greeks, cumbersome Roman number system, complicated Chinese diagrams and the like. Arithmetics based on positional decimal number notation with the use of zero was first given in it. Since mid XII c. the principles and techniques of decimal positional arithmetics the first propagandist of which in Bagdad was al-Khwarizmi gained a wide recognition in Western Europe. At that time the intense activity of a group of translators of scientific literature from Arabic into Latin (universally accepted at that time in the learned world) began to develop.

After the conquest of Spain and Portugal by Arabs (early VIII c.) the Arab-Muslim culture had found its place for long. The Arabic language was an international language of science for European intellectuals. Therefore, this period in the history of science and culture is sometimes called “Arabic”. The Spanish towns Madrid, Cordova, Toledo, Seville were cultural and scientific centres not only in the Pyrenees peninsula, but also in other states of Western Europe. “The people from neighboring France, Italy and Britain came to learn sciences from Arabs” [16, p.85].

The period of the highest development of the civilization of the peoples of Central Asia – the Oriental Renaissance coincided in time with the strongest oppression of Church

orthodoxy in all fields of European culture and science. As a result, there was a low level of mathematical knowledge and other natural sciences. Therefore, the propagation of Arab – Muslim culture in Spain played an important role in the development of the European culture including mathematics. Of leading value for mathematics here was the propagation of the achievements of Bagdad school starting from works of Muhammad al-Khwarizmi [2, p.339].

Western Europe was a successor and bearer of scientific ideas of ancient Rome and used Roman figures. Therefore, “Indian counting” did not spread here immediately. At first it was close to deep-rooted Roman number system, -counting board in letter number system. Being afraid to draw terrible accusation on themselves in heresy the scientists went to Spain secretly to learn new alien skills for them.

Prior to half XII c. Europeans “learnt Arabic sciences through the language of the original”. Of paramount importance for the progress of mathematical knowledge in Europe were as it was already said, the translations from Arabic both original writings and Greek literature as well which were in the Arabic language. Translations from Arabic were in great use in XI-XIII cc., however, the European mathematicians continued studying the Arab manuscripts in XV-XVII cc. as well. Based on the above-mentioned work of Khwarizmi there came into being algorizmic (algorithmic) tradition in the Latin West in XII-XIII cc.

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### **Analysis and Results**

A new arithmetic started to be named al - Khwarizmi in the Latinized form: “algorithm” or “algorism”, because the Latin translation of the book starts with the words: Algorizmi said (Dixit Algorizmi).

It is universally known that the Arabic original has not been found so far. The only copy of the manuscript, which was available in former times in the British abbey, was kept in the library of Cambridge University. According to the catalogue of the manuscripts, which was made up in 1858, it was rewritten in small letters (XIII c) [18, p.339].

Each algorizm (algorithm) was intended to popularize a new number system. New methods of counting with Indian figures suggested by al-Khwarizmi were much superior to the old, arduous ones. The work of al-Khwarizmi in the Latin translation served as the basis for the development of “Indian” arithmetics in Europe as well. As G.Sarton put it, this writing can be the main channel by which a new system became known in Europe [19, p.18].

The first translations of al-Khwarizmi’s works were performed by: Adelardo from Bat (1126 y.), Johan Seville (XII c.) Robert from Chester (1145 y.), Gerardo from Cremona (XII c.), Magister A. (1143 y.), Leonardo of Pisa (early XIII c.) [14, p.45].

The translation work was accompanied by making compilation writings and was of international nature: those who were involved and accepted Christianity were Arabs, Spaniards, the families who accepted Islam before, Spanish Jews, Britishers, Italians, Slavs, Flandersers. There came a great amount of literature on “algorizms” to become one of the bases of further progress of science in Europe. As M.Simon put it: “Of crucial importance in accepting in Europe decimal positional number notation and new figures arithmetics, primarily with arithmetics of al-Khwarizmi [12, p.108]. Among a number of treatizes writhen in Latin and later in living languages between XII-XV cc. of paramount interest are and play a vital role the following writings, because they are the most ancient of all known

and it is these that call al-Khwarizmi the source of inspiration... In particular, we are dealing with one of the most ancient algorisms written in XII c. and more or less dating back to the lost Arabic arithmetics of Khwarizmi. One of these treatises is available in the form of the only manuscript in Cambridge and was published in transcription in 1857 by Italian historian B. Boncompagni [3].

The author of translation is not exactly known: probably it was Herardo from Cremona. Some of the scholars assume that its author is Adelard from Bat, a British scholar and a thinker who played an important role as a pioneer of propagation of Arab science and philosophy in the first part of XII c. (J. Ruska, A.P. Yuschkevich, O. Fayzullaev, G.P. Matvievsckaya, A. Akhmedov etc.) According to Andre Allard, a scholar from Belgium who studied the first translation of the text algorism it is impossible to identify its author [5, p.55].

However, the manuscript does not represent an exact translation of al-Khwarizmi's work at all. The translation is probably a copyist of an earlier period, tailored the work of the Oriental scientist to the needs of the European reader. He substituted Arabic numerals and figures used by al-Khwarizmi and which were in use in Western Europe of that time for Roman figures.

The arithmetic treatise – algorism of Khwarizmi was revised in mid XIII c. by Spanish scholar Johann Seville. His work was called "Liber Algorismi de pratica arismetrice" (the book of Algorism on practice of arithmetics) [10].

The Latin revision is kept in Paris national library [6, p.43]. The manuscripts of J. Seville served as source for propagation of decimal positional arithmetics in Europe. Another writing often called "the Book on introduction of Alhoarism in actronomy art compiled by magister A". ("liber ysagogarum Alchorismi in artem astronomicans a magistro A".) [5, p.56] is known in two records, one of which has a date 1143. The authorship of the book is ascribed to magister A, to be considered by many scientists as Adelard from Bat, although there is no true evidence in favour of this hypothesis, according to A. Allard is not available so far. The copies of the manuscript are kept in Viena national library, in Bavaria state library in Munich, in Parisian national library and in Ambrosian library.

The development of arithmetics and mathematics at large in medieval Europe down to XVI c. was under way under the strong influence of al-Khwarizmi. It testifies that the name al-Khwarizmi in Latin transcription (algorithm) came into being in modern mathematics and the sphere of information technologies in the form of the name for calculation system performed by certain rules. A prominent scientist S. Gondz wrote: "the works of Khwarizmi – the start of the European science..." [12, p.110]. The number of works on algorism grew rapidly, they were available in various countries being first in Latin (the Latin language was the means of instruction in higher schools in Europe until XVIII c.) and later in living folk languages.

The arithmetic treatise "Algorismus demonstratus" ("The interpreted algorism") the author of which is magister Gernard (Magister Gernardus XIII c.) [13, p.403] arouse a great interest among the researchers. Also, of great publicity was "Common algorism" (Algorismus vulgaris) or "a Treatise on art of counting" (circa 1250 y.) ("Tractatus de arte numerandi") by an Englishman John Halifax [18, p.343]. The Latin nickname of this scientist – Johann Sacrobosco (Johannes de Sacrobosco, circa 1200-1256). During XIII-XIV cc. hundreds of students studied at European Universities. The professors explained the

manual on “algorism” Sacrobosco [7, p.98] in Latin – the international language of science. For centuries, this book had become the main textbook on calculations in decimal positional system of calculus in many European Universities.

Almost at the same time with algorism of Sacrobosco there came “the Interpretation of algorism” (*Demonstratio Jordani de algorismo*) by Jordan Nemorary mentioned in numerous lists [7, p.344]. Jordan Nemorary, the professor of Paris University and the General of one of the monks Orders propagated algorism in the Church monk circles.

French mathematician Alexander de Villa Dei from Normandy (died c.1240 y) set forth algorism of integer number in poetry – in 284 hexameters. His “Song on algorism (*Carmen de algorismo*)” was translated into French, English and Irish languages [7, p.345].

An anonymous French algorithmic manuscript belongs to the second half of XIII c. The ancient English algorism “The art of Counting” (“*Crafte of Nonbrynge*” c.1300 y) as well as the first famous book on algorism in the Italian language (“*Tractatus algorismi*” was drawn up by a Jacobo from Florence in 1307 [7, p.345].

An essay “A book on abacus” by Italian merchant Leonardo of Pisa or Fibonacci (1202) [18, p.342] played an important role in propagation in Europe of the positional number system. Much information taken from the mathematical works in the Arabic language was classified in it. Fibonacci is a fervent supporter of Indo – Arabic arithmetics. Since the essay was mainly intended for commercial people, it was also common among the population at large. Although the work bears the name “A book on abacus”, it has nothing to do at all with calculations using the counting board. The headline seems to be for Leonardo a term abacus as a synonym for arithmetics – in this sense it was used by Italians later.

The introduction of positional system into school education, common use of builders, engineers, calculators, calendars, traders, book-keepers, clerks and others gave rise to spread of calculations in social life, science (astronomy) and technology. The growth of algorithmic literature was largely due to invention of book-printing, some treatises were reprinted more than once. In XIV-XV cc. Indian arithmetics was widely in use among merchants and calculators (mainly in Italy and later in Germany and France).

In Germany due to growth of towns, expansion of trade and handicrafts there came a demand of business circles for learning methods of counting. It was in XIII c. that there appeared specialized schools in which would be calculators were trained. They were called *rechenmeisters* (German trade towns.) [7, p.99]. Calculators played a great role in introduction of Indo-Arabic arithmetics. The instruction was in German and in this manner the German mathematical terminology was drawn up. Soon alongside Latin there came into being numerous German textbooks on “algorism”.

Among the first and the most significant writings of such kind published in XV-XVI cc. one should name the textbooks written by Ulrich Wagner and Johann Widman. Wagner’s book entitled “*Bamberger rechenbuch*” (publ. in 1482) was the first printed German writing on “algorism”. In the book by Johann Widman (1489 y) “*Behende und Rechnung auf allen Rauffmanscaff*” only one section out of three was devoted to practical arithmetics [13, p.408]. Over three centuries the European algorithmicians had made a lot of changes and improvements in the Khwarizmi’s counting system. Particularly, in the

“Treatise on algorism” by P.Beldomandi (died in 1428) which was published in 1483, the arrangement of operations takes an up-to-date form [17, p.126].

Muhammad ibn Musa al - Khwarizmi’s Nisba<sup>i</sup> [Nisba – part of a name] in its Latin forms – more often algorithmus or algorismus turned into the name of new arithmetics, algorith or algorism<sup>ii</sup> [Orientalist J.Reino in 1849 discovered what was behind the word Nisba of Al-Khwarizmi]. Later the term algorithm was to be meant any regular computational process. Approximately, at the same time mention was made on “algorists”, (for instance, the word “algorists” is to be found in the “A book on algorism from Salem monastery i.e. adherents to algoristhetic arithmetics, the opposite to “abacusists”) [18, p.343]. This name comes from the special device – abacus which resembles present-day counting boards with the help of which all practical calculations were performed.

It should be noted that the spread of algorithmic arithmetics was not so fast (although in the middle of XIII c. new arithmetic was known both in Austria and Bavaria), and it was not due to the quality of the book and its translation but far more common reasons: the resistance of the old, strong school, i.e. abacusists, the very limitation of a group of mathematicians and dissociation, the very small number of schools, the lack of book-printing etc. Algorithmic arithmetics as from XII c started gaining its positions in a fierce struggle with abacusists who used Roman numbering and duodecimal Roman fractions and too cumbersome ways of multiplication and division.

Thus, we see that Latinized name al-Khwarizmi was entered in the title of the book and nowadays, there is no doubt that the word “algorithm” came into the European languages because of this writing. It can be said with certain, that the Uzbek scientist Muhammad al-Khwarizmi has taught Europe the art of counting and his writing on Indian arithmetics by its worthiness deserves the honour to be a handbook of the first algorithmicians of the East and West.

### **Conclusions and Suggestions**

Al-Khwarizmi’s works, as it was already said, gave rise to a new mathematical object and algorithm being widely-used in the field of informational technologies. At the outset “algorithm” meant the name of the scientist, later numbering in the positional system of counting, and now any system of calculations made according to stringent certain rules [8, p.86]. Algorithm - a regular successive process to be used for solving problems of a given type [12, p.52].

Thanks to al-Khwarizmi the European and world scientific thoughts have got acquainted with the notion algorithm, one of the basics not only in the sphere of mathematics and computer science but also in the field of informational technologies. In the XXI c. algorithm has acquired a special significance. Mathematical logic and computer science have also raised their claim to it. The present-day Internet and informational technologies cannot do without the theory of algorithms. Nowadays we know web-algorithms, algorithmic languages (Alpha, Fortran, Basic, HTML, Java etc, all in all more than 500 algorithmic languages operating in various countries worldwide), algorithms of information retrieval in the Internet. The name al-Khwarizmi has been immortalized in the very name of the scientific notion. The word “al-Khwarizmi” has developed and changed over twelve centuries and again has taken on the form “al-Khwarizmi”, but along with the

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<sup>i</sup> Nisba – part of a name.

<sup>ii</sup> Orientalist J.Reino in 1849 discovered what was behind the word Nisba of Al-Khwarizmi.

notion ‘algorithm’, al-Khwarizmi, Algorizmi, Algorismus, Alchwarism, Hwarism, Algorithm al-Khwarizmi algorithm.

The first – scientist’s name, the second - notion. It is difficult to overestimate the great service of al-Khwarizmi for world science [1]. He was a forefather of the strong cohort of scientists of the Medieval Orient who built the bridge from ancient science towards the epoch of European Renaissance. The writing of al-Khwarizmi (Khoresmi – author’s note), particularly his arithmetics served the basis both for the Oriental and for the European mathematical literature as well during the first years of the era of Renaissance. If Khwarizmi’s arithmetics was an inspiring source of any chisabu hindi in the Orient, in the West i.e. in Europe over the last period of Medieval stagnation of scientific thought and in the first years of Renaissance it performed the same role, it was Oriental school of Muhammad ibn Musa al-Khwarizmi that trained prominent mathematicians like Leonardo de Pisa nicknamed Fibonacci, Lukas du Burgo nicknamed Paccioli and others [20, p.104].

One can say with certainty that al-Khwarizmi and Central-Asian scientists in general have made their contribution to making Renaissance in Western Europe. Academician N.I.Conrad [15, p.104] was right when he said that Central-Asian culture at that time as in many fields of technology and material culture, especially in art and in the field of legislature, political doctrines, philosophy, historiography, science and fiction had developed earlier and had been richer in content than all these spheres of culture in the West.

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